Day 1 NOTES- Solving Exponential Equations

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. When solving exponential equations, you want to rewrite the equations so they have the same bases. If they have the same bases, you set the exponents equal to each other.

If $b^x = b^y$, then $x = y$

EXAMPLE 1 **Solving Exponential Equations with the Same Base**

a. $3^{x+1} = 3^5$ — Write the equation.

$x + 1 = 5$ — Equate the exponents.

$-1 \quad -1$ — Subtract 1 from each side.

$x = 4$ — Simplify.

b. $6 = 6^{2x-3}$ — Write the equation.

$1 = 2x - 3$ — Equate the exponents.

$+3 \quad +3$ — Add 3 to each side.

$4 = 2x$ — Simplify.

$\frac{4}{2} = \frac{2x}{2}$ — Divide each side by 2.

$2 = x$ — Simplify.

c. $10^{3x} = 10^{2x+3}$ — Write the equation.

$3x = 2x + 3$ — Equate the exponents.

$-2x \quad -2x$ — Subtract $2x$ from each side.

$x = 3$ — Simplify.

Solving Equations with SAME Bases

a. $3^{3x-7} = 3^{x+1}$

b. $\left(\frac{1}{2}\right)^{x} = \left(\frac{1}{2}\right)^{4x-12}$

c. $7^{3x+8} = 7^{2x-5}$

d. $2^{7x-6} = 2^{5x+2}$

e. $5^{3x-1} = 5^{3}$

f. $5^{-2x} = 5^{3x-10}$

Solving Equations with Different Bases

EXAMPLE 2 **Solving Exponential Equations with Unlike Bases**

a. $5^x = 125$ — Write the equation.
$5^x = 5^3$ — Rewrite 125 as 5^3.
$x = 3$ — Equate the exponents.

b. $4^x = 2^{x-3}$ — Write the equation.
$(2^2)^x = 2^{x-3}$ — Rewrite 4 as 2^2.
$2^{2x} = 2^{x-3}$ — Power of a Power Property
$2x = x - 3$ — Equate the exponents.
$x = -3$ — Solve for x.

c. $9^{x+2} = 27^x$ — Write the equation.
$(3^2)^{x+2} = (3^3)^x$ — Rewrite 9 as 3^2 and 27 as 3^3.
$3^{2x+4} = 3^{3x}$ — Power of a Power Property
$2x + 4 = 3x$ — Equate the exponents.
$4 = x$ — Solve for x.

When the bases are not the same, you can use the following table to help you re-write the bases so they are the same.

Bases \ Powers	2	3	4
0	0	0	0
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1000	10000

Get comfortable with using the table by rewriting the following numbers with the specified base:

a. 16 with base of 2:

b. 16 with a base of 4:

c. 125 with a base of 5:

d. 81 with a base of 3:

e. 64 with a base of 4:

Examples: Solving Equations with DIFFERENT bases

a. $2^{2m} = 16$

b. $5^{3x} = 125$

c. $3^{5x-6} = 81$

d. $2^{3x} = 4^{x+1}$

e. $6^{3x-1} = 36^{x+7}$

f. $9^{x-1} = 27^{x-4}$

h. $8^{2x} = 16^3$

j. $25^{x+2} = 625^{2x-10}$