Notes - Lengths of Circle Arcs

The distance around a circle is called the Circumference. This can be found by using this equation:

\[ C = 2\pi r \quad \text{or} \quad C = \pi d \]

Find the Circumference of the following circles: Leave your answers in terms of \( \pi \) as well as a decimal.

1) 7 cm
2) 2 yd
3) 6 km

Practice reviewing how to calculate the circumference or radius/diameter of a circle below. Leave your answers in terms of \( \pi \). Find the circumference, radius, or diameter.

A. \( r = 6 \text{ ft} \)  \hspace{1cm} B. \( d = 15 \text{ in} \)  \hspace{1cm} C. \( C = 16\pi \text{ cm} \)  \hspace{1cm} D. \( C = 40\pi \text{ m} \)
**ARC LENGTH**

**Arc Length** is a fraction of the circle’s circumference and is measured in linear units. Arc length can be calculated using the following proportion or EQUATION:

\[
\text{Arc Length} = \frac{\text{arc length}}{\text{circumference}} \times 360^\circ
\]

\[
\text{Arc Length} = \frac{m}{360} \cdot 2\pi r
\]

**WE DO:** Leave in terms of \(\pi\)

Given: \(\odot P\) and \(m \angle APC = 120^\circ\)

- Find the arc length of \(\widehat{AC}\)
  
  \[
  \text{length } \widehat{AC} = \frac{120}{360} \cdot 2\pi (4)
  \]
  
  \[
  \text{length } \widehat{AC} = \frac{1}{3} \cdot 8\pi
  \]
  
  \[
  \text{length } \widehat{AC} = \frac{8\pi}{3} \text{ units} \approx 8.4 \text{ units}
  \]

**YOU DO:** Leave in terms of \(\pi\)

- Finding the length of arc \(\widehat{AB}\)

\[
\text{Length } \widehat{AB} =
\]
Arc Measure Arc Length
\[ \text{Arc Measure} = \frac{m}{360} \times 2\pi r \]

\[ \frac{360^\circ}{360^\circ} = \frac{\text{Arc Length}}{\text{Circumference}} \]

**WE DO:** Finding the length of arc \( \overarc{BC} \)

\[ \begin{align*} 
&\text{r} = 10\text{cm} \\
&\angle A = 50^\circ \\
&\angle B = 75^\circ \\
&\text{r} = 7\text{cm} 
\end{align*} \]

**YOU DO:** Finding the length of arc \( \overarc{BC} \)
Arc Length Guided Practice

Find the arc lengths for problems 2 and 3.

1. Length of arc RS = 
   (exact answer)

2. Length of arc MN = 
   (approx. answer)

3. Length of arc AB = 
   (exact answer)

4. A circle has a radius of 6 cm. A sector has an arc length of 8.4 cm. The angle at the center of the sector is $\theta$.
   Calculate the value of $\theta$.

5. Find the radius of circle N.

6. Find the circumference of circle Q.
7. A clock has a pendulum 22 centimeters long. If it swings through an angle of 32 degrees, how far does the bottom of the pendulum travel in one swing?

For questions 8-9, use the figure below:

8) How many degrees does the minute hand move in 15 minutes? 40 minutes? 55 minutes?

9) If the minute hand is 4 inches long, what is the arc length covered by the minute hand in 40 minutes?
Skills Practice: Calculating Arc Length and Circumference

Use the diagram to find the indicated measure. Leave answers in term of $\pi$.

1. Find the circumference.

![Diagram of a circle with a radius of 7 cm]

2. Find the circumference.

![Diagram of a circle with a diameter of 20 ft]

3. Find the radius. Find the indicated measure.
   a. The exact radius of a circle with circumference 36 meters
   ![Diagram of a circle with radius $r$]
   b. The exact diameter of a circle with circumference 29 feet
   c. The exact circumference of a circle with diameter 26 inches
   d. The exact circumference of a circle with radius 15 centimeters

4. Find the length of $AB$.
   a. 
   ![Diagram of a circle with a radius of 7 in.]
   b. 
   ![Diagram of a circle with a radius of 6 ft and an angle of 120°]
   c. 
   ![Diagram of a circle with a radius of 45° and 12 cm]

5. In $\odot D$ shown below, $\angle ADC \cong \angle BDC$. Find the indicated measure
   a. $m\angle CB$
   b. $m\angle ACB$
   c. Length of $CB$
   d. Length of $ABC$
   e. $m\angle BAC$
   f. Length of $ACB$
6. Find the indicated measure.
   a. The radius of circle Q
   b. Circumference of \( \odot Q \) and Radius of \( \odot Q \)

![Diagram of a circle with angle and radius labels.]

Find the perimeter of the region. Round to the nearest hundredth.
7. 

![Diagram of a shaded region within a circle.]

8. **Birthday Cake** A birthday cake is sliced into 8 equal pieces. The arc length of one piece of cake is 6.28 inches as shown. Find the diameter of the cake.

![Birthday cake with arc length labeled.]

9. Radius = 5 in 
   Length of Arc CE = ________

10. Find the radius of the circle.
    \[ r = \]
For #11-13, solve for the requested variable. C is the center of each circle.

11. \( r = \) ________
12. \( x^\circ = \) ________
13. \( d = \) ________

14. Circumference = 10 m; Find the arc length of JT = ________

15. The arc length of OP = 10\( \pi \) inches; \( r = \) ________
16. The arc length of QT = 22 cm.; \( d = \) ________ (to the tenth)
Skills Practice – Arc Length

Practice: Find the length of each bold arc. Write your answers in terms of \( \pi \) as well as a decimal rounded to one decimal place.

1) 14 km

2) 6 mi

3) 11 yd

4) 13 mi

5) 12 yd

6) 13 m

7) 17 in

8) 16 in
**Practice:** Find the length of each bold arc. Write your answers in terms of $\pi$ as well as a decimal rounded to one decimal place.

Find the length of each arc.

1) \[\text{90°} \quad 8 \text{ km}\]

2) \[\text{18 cm} \quad \text{45°}\]

3) \[\text{270°} \quad 11 \text{ yd}\]

4) \[\text{3 ft} \quad \text{315°}\]

5) \[\text{16 yd} \quad \text{315°}\]

6) \[\text{15 ft} \quad \text{135°}\]

7) \[\text{4 ft} \quad 45°\]

8) \[\text{13 m} \quad 225°\]